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Technology, Gender Inequality, and Fertility

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Abstract

This paper proposes a new mechanism linking technology, gender inequality in education, and fertility in a unified growth model. There are three main components to the mechanism: First, increases in the level of technology not only increase the return to human capital but also reduce women's time in doing housework, leaving women with more time for child care and labor-force participation, since technological progress creates labour-saving products for doing housework. Second, the decreases in women's time devoted to housework in the future make households today invest less in education for their sons in order to invest more education for their daughters because the marginal return to female education is higher than that to male education, therefore, improving the gender equality in education. Third, the better gender equality in education, in turn, accelerates the technological progress. This positive feedback loop generates a demographic transition accompanied with accelerated economic growth.

Keywords: Technological level, technological progress, gender inequality in education, fertility, human capital.

JEL Classification: J11, J13, J16, O11, O40.

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1. Introduction

Recently, there has been a renewed interest, both from theoretical and empirical viewpoints, in the relationships between gender inequality, fertility and growth to explain some stylized facts during development processes of societies, such as Galor and Weil (1996), Klasen (2002), Lagerlof (2003), Klasen and Lamanna (2009), Doepke and Tertilt (2009), De la Croix and Vander Donckt (2010), Diebolt and Perrin (2013a, b), and others. The stylized facts observed widely across societies are: (i) a negative correlation between fertility and female-to-male education ratio; (ii) a positive correlation between per-capita income and female-to-male education ratio; (iii) a negative correlation between female-to-male earning ratio and fertility; (iv) a decline over time in the human capital gender gap and earning gap between male and female workers; (v) an increase over time in the labor-force participation of women, and (vi) a demographic transition of societies entering the regime of modern sustained economic growth.

Gender gaps versus fertility and per-capita income growth

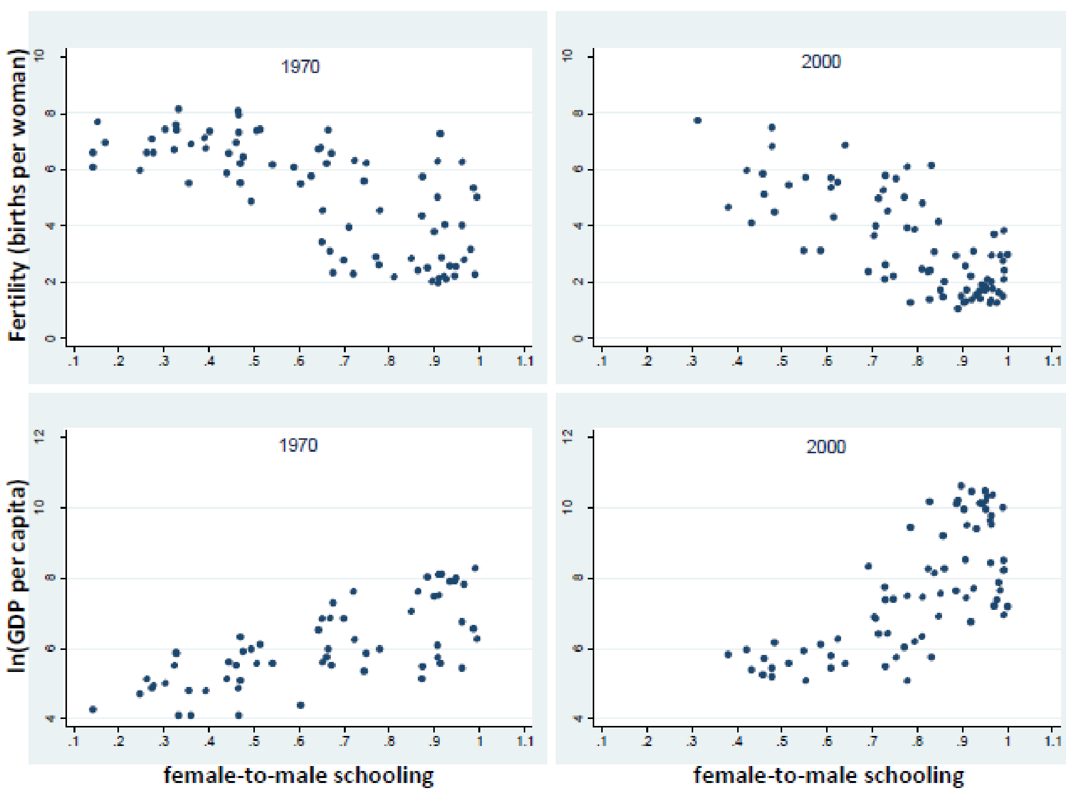

Figure 1. Cross-country plots of fertility and per-capita income against gender equality in education. Source: World Bank (2013).

The Figure 1 above provides showing the cross-79-country picture of per-capita income (in logarithms, US dollar), and the fertility rate, against gender inequality in education in years 1970 and 2000 which is measured by the ratio of the number of schooling years of women over that of men. The gender inequality in education looks strongly positively correlated with per-capita income, and negatively correlated with fertility.

The strong negative correlation between gender inequality in earning and fertility is showed from the dynamics of gender gap in earning and fertility in Sweden from 1870 to 1924. One can argue that the gap in earnings between men and women is due to the gap in education between them. So the Figure 2 below may reflect the upper graphs in Figure 1 above.

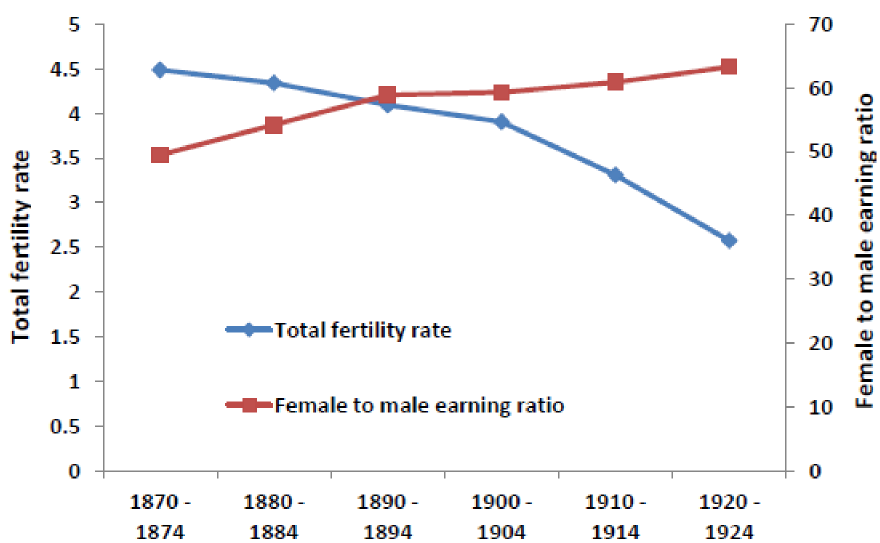

Figure 2. Female relative wages and fertility rates in Sweden 1870–1924. Source: Schultz (1985)

The negative correlation between gender inequality in income and fertility has been well explained in the literature, such as in Galor and Weil (1996), Lagerlof (2003), and others. Like these previous research, we argue in this paper that an increase in female relative wage increases the opportunity cost of raising children more than household income, which makes fertility decline (when the level of technology is sufficiently high). However, this paper re-explains this stylized fact in a different unified growth framework with a different mechanism leading to a decline in gender inequality in income. Galor and Weil (1996) argue the decline in the gender wage gap over time to be due to the accumulation of physical capital, because physical capital is more complementary to women's labor than men's. Lagerlof (2003) treats the relative human capital gap between women and men, and hence relative wage gap between them, as exogenous (by fixing this relative gap) and considers impacts of the different relative gender wage gaps on the divergence in fertility and the long run growth across societies. We show in this paper that the technological progress makes the gender relative gap in human capital decline, and hence improves the gender equality in relative income.

Gender gap in human capital

The figure 3 below uses literacy rates to stand for human capital. The literacy rate is not a unique measurement for human capital. However, the evolution of these rates for men and women can be a proxy for the evolution of human capital, in particular it gives an intuition for the decline in the gender gap in education as well as human capital.

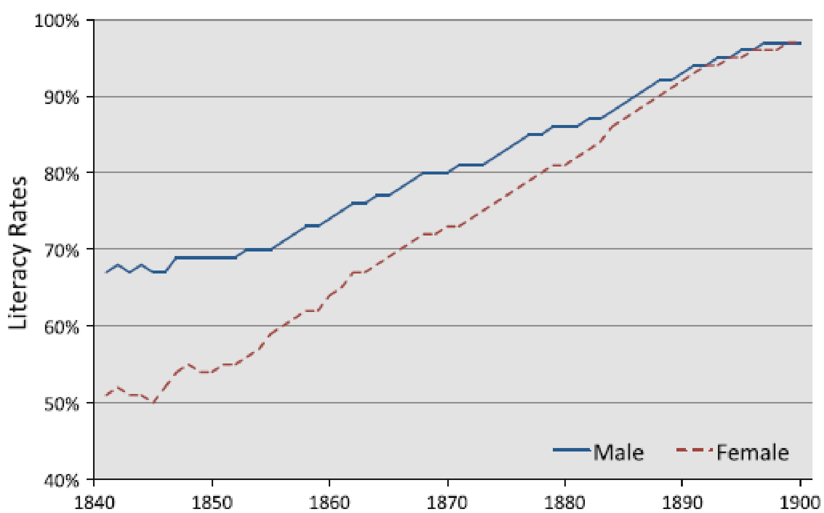

Figure 3. The decline in human capital gap: England 1840 - 1900. Source:[2] Cipolla (1969)

Most of papers in the related literature study the effects of gender inequality in education or human capital on economic growth, as well as the effect of economic growth on gender inequality in education. A list of selected papers includes Barro and Lee (1994), Dollar and Gatti (1999), Klasen (2002), Lagerlof (2003), and Klasen and Lamanna (2009), etc. However, a question should be addressed here is why gender inequality in human capital decreases during the development process? And how this dynamics of human capital relates to other stylized facts?

[2] Quoted in Galor (2012)

Labor-force participation of women

The Figure 4 below shows the increase over time in female labor-force participation along with the decrease in fertility in OECD countries.

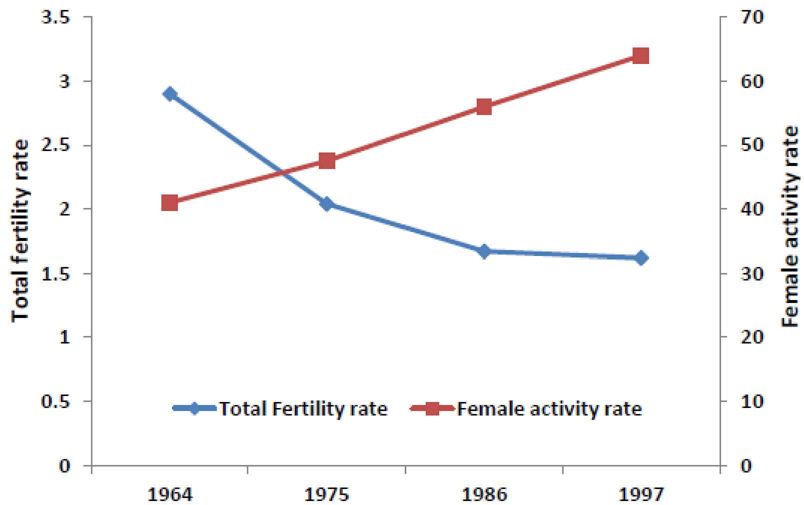

Figure 4. Fertility and female activity rate in OECD countries. Source: Adserà (2004).

The increase over time in female labor-force participation along with the decrease in fertility due to decreases in the gender wage gap between female and male is examined theoretically in Galor and Weil (1996), Lagerlof (2003), and Bloom et al. (2009). Greenwood et al. (2005) explain the increase in female labor-force participation by technological progress creating labor-saving products for doing housework in a model with constant fertility. This paper is based on the impact of technological progress on women's time devoted to housework as proposed in Greenwood et al. (2005). However, this paper differs from Greenwood et al. (2005) is that it endogenizes technological progress and analyzes the improvement in gender inequality in education due to the impact of technological progress on women's time devoted to housework, while explaining the demographic transition during the development process.

Evolution of income growth and population growth in Western Europe

The population and output of Western Europe experienced three distinct regimes during the last couple of millenia: from Malthusian stagnation, through the demographic transition, to modern sustained growth. After thousands of years in Malthusian stagnation characterized by very low growth rates in both per-capita income and population, the economy then entered in the the phase of demographic transition in which both growth rates of per-capita income and population increased simultaneously. And later on, the growth rate of per-capita income still increased while the growth rate of population fell, the economy entered the regime of modern sustained growth (see figure 5). This stylized fact, indeed, is explained by Galor and Weil (2000) and Galor and Moav (2002) in unified growth models, and recently confirmed empirically by Becker et al. (2010, 2011). Like these papers, this paper also highlights the role of human capital on the technological progress and the demographic transition and the interactions between these issues during the development process. These papers, however, ignore the gender issue and its interactions with technology and fertility, while this paper does.

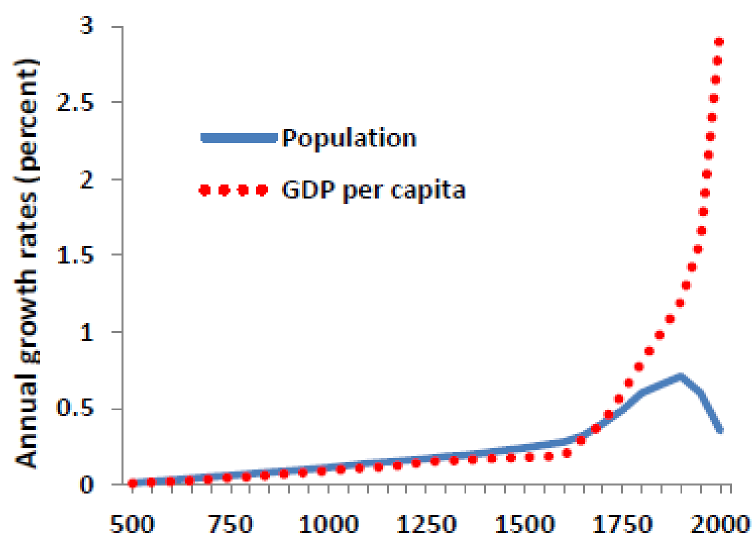

Figure 5. The growth rates of per-capita income and population of Western Europe in the three regimes. Source: Madison (2001)

The closest literature to this paper may be the paper of Diebolt and Perrin (2013b), which also proposes a unified growth model to explain the development process considering gender inequality. Basically, Diebolt and Perrin (2013b) also stress the importance of human capital accumulation for economic growth and the positive effect of technological progress on skilled human capital through an increase in the return to education. The most important differences between Diebolt and Perrin (2013b) and this paper, which leads to different explanations for development process from stagnation to modern sustained growth, are: (i) Diebolt and Perrin (2013b) assume that individuals invest education for themselves when they are adults, while in this paper we assume that individuals receive educational investment from their parents when they were children; (ii) Diebolt and Perrin (2013b) consider the *bargaining power* of the wife, which depends on both incomes of the wife and the husband, determining the equality in human capital between the wife and the husband, while this paper considers the *positive effect of technological progress on the possible female labor supply* which makes the households increase the share in educational investment for their daughters. Therefore, the mechanism for the transition from stagnation to modern sustained economic growth in Diebolt and Perrin (2013b) differs from the one proposed in this paper. In their model, over time, technological progress triggers the female empowerment which induces women to invest more in human capital for themselves, contributing to human capital accumulation, and hence fostering economic growth. In parallel, the higher female human capital increases the opportunity cost of raising children, making the fertility decline. The mechanism in this paper, however, is that the technological progress increases the possible female labor supply which makes the households increase the share in educational investment for their daughters. The better equality in human capital between women and men, in turn, accelerates technological progress. These feedback interactions also generate both demographic and economic transitions. However, in Diebolt and Perrin (2013b), the growth rate of population, proxied by fertility, always declines over time along with technological progress and bargaining power of the wife, while it is a fact, which is *captured* in this paper, that the growth rate of population increases during the early stages of development (before the demographic transition). And because Diebolt and Perrin (2013b) assume that adult individuals spend time for educating themselves to increase their human capital, then although their model generates the decline over time in fertility with the increase in human capital, the explanation for the increase over time in female labor-force participation (corresponding to the stylized fact depicted in the Figure 4) is still absent. In addition, Diebolt and Perrin (2013b) assume the human capital formation is linear in education investment (for given other factors, the marginal return to education investment is always constant when education investment exceeds a fixed cost), while we assume in this paper that the human capital formation is an increasing and concave function of education investment. Therefore, the positive effect of gender equality in education on technological progress comes from a positive externality of women's human capital on their children's human capital formation, while in our model this positive effect comes from the higher marginal return to female education in human capital formation.

Although there has been a huge empirical literature considering the relationship between gender inequality and economic growth, the theoretical literature on this issue seems rather limited. In addition, to the best of my knowledge, no published paper so far has explained all stylized facts above in one single theoretical model. Most papers in the related literature only explain the combinations of some of the stylized facts above. This paper aims at contributing a simple unified growth model capturing technological progress, gender inequality in education, fertility and the complex interaction between these issues to explain all stylized facts listed above. This paper shows that the demographic transition to modern sustained growth, the decline over time in the human capital and earning gender gaps, and the increase over time of the labor-force participation of women are inevitable outcomes of the development process when the driving force for technological progress is the average human capital.

The rest of paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 analyses the effects of technological progress on gender inequality in education, fertility, and female labor-force participation. The competitive equilibrium and the dynamical system are identified in section 5. Section 6 analyses the development process to explain the stylized facts. Section 7 concludes the paper.

2. Related literature

Cubers and Reignite (2012) provide an excellent review on gender inequality and economic growth both in theory and empirics. They consider gender inequality in its many aspects. From their review we find that, most papers in the literature conclude that gender inequality is harmful for economic growth, and economic growth help to improve gender equality.

Among previous theoretical works considering gender inequality in growth models, the most cited may be Galor and Weil (1996). The authors advance a theory to interpret the loop relationship among gender gap, fertility and growth in which higher wages of women reduce fertility by raising the opportunity cost of children. The lower fertility, in turn, raises the level of per capita physical capital. In Galor and Weil (1996), the only difference between men and women, resulting in gender wage gap, is that men have more physical strength than women. Physical capital is more complementary to mental labor than to physical labor, so women's wage is then increased relatively to that of men due to capital accumulation. The model exhibits multiple steady state equilibria, one in which fertility is high while output and capital per worker are low, and hence women's relative wage is low. The other is characterized by low fertility, high output and capital per worker, hence high women's relative wage. They conclude that countries with a high initial level of capital per worker will converge to a high income level equilibrium with low fertility and high relative wages for women. The opposite would be true for countries with a low initial level of capital per worker. Cavalcanti and Tavares (2007) use Galor and Weil (1996) model, considering for the size of government, to argue that the increase in income per capita and decline in fertility are accompanied by two changes in structure: (i) an increase in the share of government expenditure in total output; and (ii) an increase in women's labor force participation. These two changes are causally related. The approach in Galor and Weil (1996), however, assume that women and men have identical human capital implying male and female receive the same educational investment, whereas differences in education are observed widely in many countries with females typically receiving less education than males.

Lagerlof (2003) examines the links between gender inequality in human capital and long run economic growth. The author points out a threshold of relative equality in human capital between women and men beyond which (i.e. for relatively high equality) the economy can exhibit sustained growth, otherwise the economy converges to a non-growing steady state, a Malthusian trap. The paper of Lagerlof (2003) implies that inequality in human capital can result in high fertility, low economic growth, and continued gender inequality in providing human capital for males and females, thus generating a poverty trap that calls for public intervention. The paper of Lagerlof, however, assumes that (for whatever reason) men have more human capital than women and the relative inequality in human capital between them is fixed, i.e. it treats this relative inequality as an exogenous variable. In fact, gender inequality may derive from culture, and its size may change endogenously along with technological progress.

Greenwood et al. (2005) provide a channel through which economic growth affects positively gender equality in employment. They argue that technological progress in the household sector is embodied in the form of new labor-saving consumer durables which free up women's time devoted to housework, making them to increase their labor force participation. In this approach, however, the authors explicitly assume that the technological progress and gender gap are exogenous, and ignore the fertility factor as well as education investment.

Doepke and Tertilt (2009) propose an interesting mechanism of a positive effect of growth on gender equality. The authors investigate men's incentive to share power with their wives. They argue that, from a man's perspective, he wants his wife to have no rights. But he cares about his daughters's marital power bargaining vis-à-vis his son-in-law because he is altruistic to his children. In Doepke and Tertilt (2009), an expansion of a wife's legal rights increases human capital investment for her children, helping them to find quality spouses which are also in the preference of their father. Therefore, the father gains from the increasing power of his children's future mothers-in-law because his children will have quality spouses. That is to say men face a trade-off between the rights they want for their own wives and the rights of other women in society. This trade-off shifts over time because of the changing role of human capital driven by technological progress. The authors show that when the returns to education are low, men will vote for the regime in which all family decisions are made solely by the husband. When technological progress changes the importance of human capital men may vote for the regime under which decisions are made jointly by husband and wife. De la Croix and Vander Donckt (2010) propose a model, which is also based on the intrahousehold bargaining between man and woman, capturing several aspects of gender inequality (such as survival gap, wage gap, social and institutional gap, and educational gap) to analyze their impacts on demographic and economic outcomes for least developed countries. The authors point out the key measures to ease these countries out of poverty trap are to promote survival probabilities of female and infant, which make women more likely active in the market, leading to female education to be more important. A better female education increases the bargaining power of women in the households' decision process, hence decreasing fertility and improving the quality of children, as well as fosters economic growth. One can find more analytical works relating to woman's rights and marital power bargaining in Basu (2006), Fernandez (2009), Doepke and Tertilt (2011), Doepke et al. (2012), and more recently Diebolt and Perrin (2013a, b).

In parallel to theoretical studies, a huge empirical literature has also examined the complex relationship between gender inequality and economic growth. The availability of comprehensive international datasets has allowed the emergence of a large number of time series, cross-section, and panel data empirical studies of this topic. An early study by Barro and Lee (1994) reported a "puzzling" finding that gender inequality in education might increase economic growth. The authors find that when they include male and female primary and secondary schooling in regression, the coefficient associated with female schooling is negative. However, more recent papers have shown the opposite appears, i.e. gender inequality in education reduces economic growth

(Dollar and Gatti 1999, Forbes 2000, Knowles et al. 2002, Klasen 2002, Abu-Ghaida and Klasen 2004, Klasen and Lamanna 2009). These papers also explain why Barro and Lee (1994) found the opposite effect and the more careful econometric techniques yield the finding that gender inequality in education inhibits economic growth.[3]

Dollar and Gatti (1999) study the effects of gender gaps in education, health and life expectancy, the legal and economic equality in society and marriage, and degree of women's empowerment on economic growth. In contrast to Barro and Lee (1994), they find that the coefficient associated with female education is positive, whereas that associated with male education is negative but statistically insignificant. However, the positive effect of female education on growth is nonlinear: increase in female education has no effect on economic growth for countries with very low female education. But, in countries with relatively high female education, increasing it spurs economic growth. Dollar and Gatti also provide a strong evidence that increases in per capital income lead to improvements in gender equality in education and health care. And, they conclude that societies that have a preference for not investing in girls pay a price for it in terms of slower growth and lower income.

By using cross-country data 1960 - 2000 and panel regression, Klasen (2002) and Klasen and Lamanna (2009) show that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment. In contrast to Barro and Lee (1994), they find that the negative coefficient associated with female education disappears when the multicolinearity problems are taken into account and dummy variables of regions are added. Interestingly, these two papers estimate lower and upper bounds of the effect of gender inequality on economic growth. The findings of these papers differ significantly from Dollar and Gatti (1999) in the point that they find that the negative effect of gender inequality in education not only appears among countries with relative high female education, but appears among countries with low female education also. They justify the causes for this difference by the fact that they use a larger time-series dataset (1960 - 2000 rather than 1975 - 1990). They use a different measure of human capital (the total year of schooling rather than the share of the adult population with secondary education). And they claim that the multicolinearity problem were ignored in Dollar and Gatti (1999).

[3] Many authors show that Barro and Lee (1994) identified the absence of regional dummy variables, particularly for Latin America and Ease Asia, making their estimation biased. Their biased finding may also related to multicolinearity. In most countries, female and male education are closely correlated, making it difficult to estimate their individual effects. Large standard errors for male and female education in Barro and Lee (1994) and the sudden reversal of this finding in other specifications is a strong evidence of this problem. For more discussion of these issue, see Dollar and Gatti (1999), Forbes (2000), Klasen (2002), and Klasen and Lamanna (2009).

Along with these empirical studies above, many papers also find out a negative effect of gender inequality in education on economic growth. An incomplete list of papers includes Hill and King (1995), Tzannatos (1999), Lorgelly and Owen (1999), Forbes (2000), Knowles et al. (2002), and Abu-Ghaida and Klasen (2004), etc.

3. The model

We consider a two-sex overlapping generations economy in which people live for two periods. In the first period, they are children and they are raised as well as provided educational investment by their parents. In the second period, they are adults, getting married to be households. The representative agent hereafter is household (or couple). In the adulthood period, agents are endowed one unit of time to raise children, to do housework, and to supply their labor to market to earn income for their consumption and educational investment for their children.

3.1. Preferences and constraints

In any period $t \in \mathbb{N}$, the economy consists of L_t identical households. Each household is composed by one man and one woman. Each member in household is endowed one unit of time. Households allocate their time resources for labor supply to market to earn income, for child-rearing, and for housework. As in Becker (1985, p52), we assume that in a household, the woman is fully responsible for child-rearing and doing other housework.[4] The time devoted to raising children and doing housework cannot be used to work in the market. We use indexes m and f to denote for sexes male and female respectively. Households born at date $t-1$ have preferences over their consumptions at date t, the number of their children, and the income that their children can earn when they are adults as follows

$$u_t = \gamma \ln \left(n_t w_{t+1} \left[h_{t+1}^m + [1 - \varphi(A_{t+1})] h_{t+1}^f \right] \right) + (1-\gamma) \ln c_t \tag{1}$$

where c_t is consumption of household in period t; n_t is the number of children (since the basic unit of analysis in this model is a couple then n_t is in fact the number of couples

[4]One may think that working in the market requires not only human capital but physical strength of workers also. So men have more advantage to supply their labor to the market than women have. One may also justify that for some psychological reasons as well as conventions of societies, women are supposed to be responsible for child care and other housework. In fact, introducing time for men to do housework does not change the qualitative analysis as long as men take care of the housework less than women do.

of children that each couple t has)[5]; w_{t+1} is the return to human capital in period $t+1$; h^m_{t+1} and h^f_{t+1} are human capital of each male and female children respectively; A_{t+1} is the level of technology in period $t+1$; and $\varphi(A_{t+1})$ is the time that women will spend for housework in period $t+1$.

We assume here that the time for doing housework is *inevitable* and households in period t perfectly foresee that their daughters will spend that time doing housework in the period $t+1$, while households do not have perfect foresight on the fertility of their offspring. Therefore, households consider $w_{t+1}\left[h^m_{t+1} + [1-\varphi(A_{t+1})]h^f_{t+1}\right]$ as the potential income that each couple of their offspring can earn when they are adults.

Greenwood et al. (2005) provide a theoretical framework to argue that technological advancements in the household sector play crucial role in liberating women from housework. It is obvious that, along with technological progress, it is the appearance of household sector products such as washing machines, vacuum cleaners, refrigerators, etc. which help women save time in doing housework. The appearance of some other products, such as frozen foods and ready made clothes, due to technological progress also liberate women from housework. So, it is rather plausible to assume that the time devoted to housework is decreasing in the level of technology. Hence, we assume that[6]

$$\varphi'(A) < 0,\, \varphi''(A) > 0 \qquad \text{and} \qquad 0 = \lim_{A\to+\infty}\varphi(A) < \bar{\varphi} = \varphi(0) < 1 \qquad (A1)$$

Human capital formation for a child with sex $i \in \{m, f\}$ is

$$h^i_{t+1} = (e^i_{t+1})^\theta \tag{2}$$

where $\theta \in [\frac{1}{2}, 1)$ for a reason to be apparent in section 4.2; e^i_{t+1} is educational investment for a child with sex $i \in \{m, f\}$.

The budget constraint of the household born at date $t-1$ is

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) \le w_t(h^m_t + [1-\varphi(A_t)]h^f_t) \tag{3}$$

where ρ is the cost in time required to raise one couple of children physically. Since only the wife takes care children in the household and the time raising children cannot be used to work in the market, then the opportunity cost for raising one couple of

[5]Here we also assume that the gender birth ratio (male over female) is $1:1$ which is closed to the natural gender ratio $1.05:1$.

[6]The assumption $\lim_{A\to+\infty}\varphi(A) = 0$ is a simplification. The analysis does not change qualitatively if we set $\lim_{A\to+\infty}\varphi(A) = \underline{\varphi} \in (0, \bar{\varphi})$.

children is $\rho w_t h_t^f$.

Since each person in period t is endowed one unit of time, then the time constraint of the woman in period t is

$$\rho n_t + \varphi(A_t) \leq 1 \tag{4}$$

3.2. Production and technology

In each period t, output can be produced out of human capital according to the following production function

$$Y_t = f(A_t)H_t \tag{5}$$

where Y_t is aggregate output produced in period t; H_t is aggregate human capital supplied to production in period t; $f(A_t) > 0$, $f'(A_t) > 0$ $\forall A_t > 0$, and A_t is the level of technology in period t;[7] and $\lim_{A_t \to +\infty} f(A_t) = \bar{f}$.

The output per household in period t is

$$y_t = \frac{Y_t}{L_t} = f(A_t)\left(h_t^m + [1 - \rho n_t - \varphi(A_t)]h_t^f\right) \tag{6}$$

The return to human capital in period t is

$$w_t = f(A_t) \tag{7}$$

The dynamics of technology is

$$A_{t+1} = (1 + g_t)A_t \tag{8}$$

where g_t is the rate of technological progress between periods t and $t+1$. We assume that g_t depends on the average human capital of the working generation t, i.e.

$$g_t = g\left(\frac{h_t^m + h_t^f}{2}\right) \tag{9}$$

where $g(h) > 0$, $g'(h) > 0$, $\forall h \geq 0$.

[7]Here we simplify the production function by ignoring the role of physical capital to focus on the role of human capital. One can introduce physical capital and extend each agent's life to three periods, and hence the agents allocate their resources not only for consuming and educating their children, but also for saving for consumption when old. However, the conclusions of paper do not change qualitatively.

3.3. Household's optimization

Households of generation t choose the optimal mixture of quantity and quality of their children and supply their remaining time, after finishing housework, in the labor market to consume their wages so as to maximize their inter-temporal utility function. Substituting (2) into (1), the optimization of the representative household is

$$\max_{\substack{c_t, n_t > 0 \\ e^m_{t+1}, e^f_{t+1} \geq 0}} \gamma \ln \left(n_t w_{t+1} \left[(e^m_{t+1})^\theta + [1-\varphi(A_{t+1})](e^f_{t+1})^\theta \right] \right) + (1-\gamma) \ln c_t \tag{10}$$

subject to

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) \leq w_t(h^m_t + [1-\varphi(A_t)]h^f_t) \tag{11}$$

$$\rho n_t + \varphi(A_t) \leq 1 \tag{12}$$

for given w_t, h^m_t, h^f_t, A_t, perfectly foreseen w_{t+1}, A_{t+1}, and parameters γ, θ, ρ.

Solving this problem (see Appendix A1 and A2), we get the optimal choice of a household

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t)\right) & \text{if} \quad \gamma(1-\theta)\left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t)\right) + \varphi(A_t) < 1 \\ \\ \frac{1-\varphi(A_t)}{\rho} & \text{if} \quad \gamma(1-\theta)\left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t)\right) + \varphi(A_t) \geq 1 \end{cases} \tag{13}$$

$$c_t = \begin{cases} (1-\gamma)w_t(h^m_t + [1-\varphi(A_t)]h^f_t) & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{(1-\gamma)w_t h^m_t}{1-\gamma+\gamma\theta} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{14}$$

$$e^m_{t+1} = \begin{cases} \frac{\theta\rho w_t h^f_t}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{15}$$

$$
e_{t+1}^f = \begin{cases} \frac{\theta \rho w_t h_t^f}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{16}
$$

From (13) we find that when the time constraint of the wife is not binding, then the fertility is increasing in the human capital gap between the husband and the wife. In the case the time constraint of the women is not binding, consumption of the household is increasing linearly in the potential income of household (abstracting the opportunity costs for doing housework) and educational investment for each child are increasing linearly in the potential income of the women. While when the time constraint of the women is binding, i.e. the women spends full time for child-rearing and doing housework, then the consumption of the household, and educational investment for male and female children are increasing linearly in the real income of the household, i.e. the income of the husband.

4. Gender inequality and fertility with technology

Before examining the dynamical system, it is interesting to analyse the impact of technological progress on gender inequality in education and fertility. These analyses help us to understand better the simultaneous evolution of gender inequality in education and fertility along with technology.

4.1. Gender inequality in education

We define the following measure of gender inequality (female over male) in education in period t

$$
\mu_t = \frac{e_t^f}{e_t^m} \tag{17}
$$

where (17) implies that a complete equality in education between genders in period t appears when $\mu_t = 1$, and an education bias toward male (female) when $\mu_t < (>)1$.

From (15) and (16), we have

$$
\mu_{t+1} = \frac{1 + [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}}}{1 + [1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}}} = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} \equiv \mu(A_{t+1}) < 1 \tag{18}
$$

$$
\mu'(A_{t+1}) = \frac{1}{\theta-1}[1-\varphi(A_{t+1})]^{\frac{\theta}{1-\theta}}\varphi'(A_{t+1}) > 0 \tag{19}
$$

$$\mu''(A_{t+1}) = \frac{1}{\theta-1}\left[\frac{\theta}{1-\theta}[1-\varphi(A_{t+1})]^{\frac{2\theta-1}{1-\theta}}\varphi'(A_{t+1})^2 + [1-\varphi(A_{t+1})]^{\frac{\theta}{1-\theta}}\varphi''(A_{t+1})\right] < 0 \tag{20}$$

So the gender inequality in education is strictly increasing and concave in the level of technology. Under the assumption $(A1)$ we have

$$\lim_{A_{t+1}\to+\infty}\mu(A_{t+1}) = 1 \quad \text{and} \quad \lim_{A_{t+1}\to 0^+}\mu(A_{t+1}) = (1-\bar{\varphi})^{\frac{1}{1-\theta}}$$

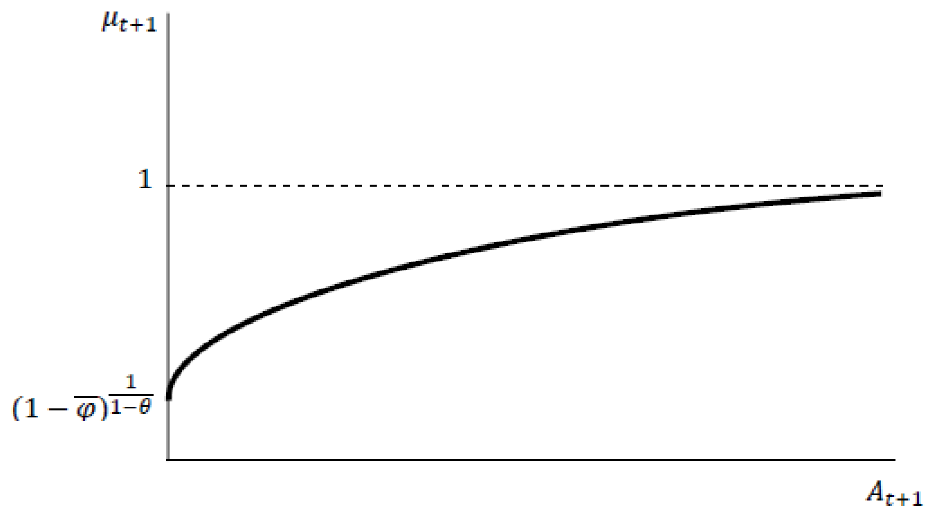

Figure 6. Gender inequality in education against technology

From (15) and (16), we also have

$$e^m_{t+1} + e^f_{t+1} = \begin{cases} \frac{\theta\rho w_t h^f_t}{1-\theta} & \text{if } \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)]} & \text{if } \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{21}$$

From (18) we find that the gender inequality in education is always biased towards males, i.e. male children receive more educational investment than female children do. This is because when children become adults in the period $t+1$, the women have to spend a fraction of time to do housework while men do not. The time devoted to housework cannot earn income. So economically, for a given amount for education investment per one couple of children as in (21), a rational household in period t invests less education on their daughter and invest more on their son. Interestingly, the inequality in education is decreasing in the level of technology in period $t+1$. The reason is rather intuitive when higher level of technology in period $t+1$ reduces the time that women devote to housework then they have a chance to increase their participation to the labor force, leading their parents in period t, by utility optimization behavior, to invest more education for their daughter to increase labor productivity. As a result, educational investments for male children decrease. Consequently, the inequality in education decreases along with the increase in technology.

Now it is interesting to examine the impact of gender inequality in education on the growth rate of technological progress.

Proposition 1: *In the overlapping generations economy above, in any period t, the better the gender equality in education, the higher the growth rate of technological progress. This also implies that the growth rate of technological progress, g_t, gets maximum when complete gender equality in education prevails.*

Proof: In effect, from (21) with one period lagged, let us denote

$$\Sigma_t = e_t^m + e_t^f$$

which is determined in period $t-1$ and independent of the gender inequality in period t.

The growth rate of technology between period t and $t+1$ is defined in (9), i.e.

$$g_t = g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2}\right) = g\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)$$

so that

$$\frac{\partial g_t}{\partial e_t^f} = g'\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)\frac{\theta}{2}\left[(e_t^f)^{\theta-1} - (\Sigma_t - e_t^f)^{\theta-1}\right]$$

Since $g'(\cdot) > 0$ and $\frac{1}{2} \leq \theta < 1$ then

$$\frac{\partial g_t}{\partial e_t^f}\begin{cases} > 0 & \Leftrightarrow \quad e_t^f < \Sigma_t/2 \\ = 0 & \Leftrightarrow \quad e_t^f = \Sigma_t/2 \\ < 0 & \Leftrightarrow \quad e_t^f > \Sigma_t/2 \end{cases} \tag{22}$$

Since from (17) and (18), it holds

$$\mu_t = \frac{e_t^f}{\Sigma_t - e_t^f} < 1 \quad \forall t \quad \Rightarrow \quad e_t^f < \Sigma_t/2 \quad \forall t$$

So an increase in education of women e_t^f (when $e_t^f < \Sigma_t/2$), i.e. a better gender equality in education, leads to a higher growth rate of technological progress. And (22) also implies that in the period t, the growth rate of technological progress g_t is maximum when a complete gender equality in education prevails, i.e. $\mu_t = 1$.

Q.E.D.

Proposition 1 is consistent with empirical results Klasen (2002) showing a positive effect of gender equality in education on economic growth. Indeed, Proposition 1 shows a positive effect of gender equality in education on the growth rate of technology which, in turn, affects positively the income growth. This is because in any period t the marginal return to education is higher for the female children than for male children. Hence, for a given amount of educational investment for children, investing more on female children until complete gender equality appears would increase the average human capital for the economy. As a result, the higher the average human capital, the higher the growth rate of technological progress. The empirical evidence for the statement in Proposition 1 and its mechanism can be found in Klasen (2002).[8]

4.2. Fertility and labor-force participation of women

The equation (18) still holds when we step back one period, i.e.

$$\mu_t = \frac{e_t^f}{e_t^m} = [1 - \varphi(A_t)]^{\frac{1}{1-\theta}} \tag{23}$$

so that

$$\frac{h_t^m}{h_t^f} = \frac{(e_t^m)^\theta}{(e_t^f)^\theta} = [1 - \varphi(A_t)]^{\frac{\theta}{\theta-1}} \tag{24}$$

Substituting (24) into (13), we have

[8] Klasen (2002) shows that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment.

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) \equiv n_a(A_t) \text{ if } n_a(A_t) \leq \frac{1-\varphi(A_t)}{\rho} \\ \\ \frac{1-\varphi(A_t)}{\rho} \equiv n_b(A_t) \qquad\qquad\qquad\qquad \text{if } n_a(A_t) \geq \frac{1-\varphi(A_t)}{\rho} \end{cases} \tag{25}$$

For the existence of a solution to $n_a(A_t) = n_b(A_t)$, i.e. the existence of a threshold for the technological level below which women will spend full time doing housework and raising children, we assume that

$$\bar{\varphi} > 1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta} \qquad (A2)$$

The assumption $A2$ requires that at some low level of technology, the time devoted to housework of women is sufficiently high to make them not to supply their labor to the market. If this assumption did not hold, the time constraint of women would be never binding for all $A_t > 0$, as derived from (25). That is to say, women would always supply their labor to the market regardless how low the strictly positive contemporary level of technology is, and the fertility would be always decreasing in the level of technology. Nevertheless, in the early stages of development, fertility is typically observed to be increasing with the level of technology, and women supply their labor to the market when the return to their labor was sufficiently high.

Lemma 1: *Under assumptions* $A1$ *and* $A2$*, there exists a unique* $A^* > 0$ *such that*

$$n_a(A^*) = n_b(A^*)$$

Proof: In effect we consider the equation

$$n_a(A_t) = n_b(A_t)$$

that is to say

$$\gamma(1-\theta)\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) = 1 - \varphi(A_t)$$

or equivalently

$$\varphi(A_t) = 1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}$$

Under assumption $A1$, $\varphi(A_t)$ is an invertible function. Thus, under assumption $A2$ there exists a unique $A^* > 0$ solving $n_a(A_t) = n_b(A_t)$, where

$$A^* = \varphi^{-1}\left(1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}\right)$$

Q.E.D.

So, under Lemma 1, we can rewrite (25) as follows

$$n_t = n(A_t) = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) \equiv n_a(A_t) \text{ if } A_t \geq A^* \\ \\ \frac{1-\varphi(A_t)}{\rho} \equiv n_b(A_t) \qquad\qquad\qquad\qquad\qquad\qquad \text{if } A_t \leq A^* \end{cases} \tag{26}$$

Equation (26) implies that in any period t women participate in the labor market if, and only if, the contemporary level of technology A_t is sufficiently high (i.e. $A_t > A^*$), otherwise women will spend their full time doing housework and raising children. So, A^* is thus the highest level of technology for which the women do not work in the market.

To see the impact of technology on female participation in the labor market, not first that $\forall A_t$,

$$n'_a(A_t) = \frac{\gamma(1-\theta)}{\rho}\left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}} - 1\right)\varphi'(A_t) < 0$$

since $\theta \in [\frac{1}{2}, 1)$ as mentioned in section 3.1.

$$n''_a(A_t) = \frac{\gamma(1-\theta)}{\rho}\left[\frac{\theta\varphi'(A_t)^2}{(1-\theta)^2}[1-\varphi(A_t)]^{\frac{2-\theta}{\theta-1}} + \left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}} - 1\right)\varphi''(A_t)\right] > 0$$

$$\lim_{A_t \to +\infty} n_a(A_t) = \frac{2\gamma(1-\theta)}{\rho} \quad \text{and} \quad n_a(0) = \frac{\gamma(1-\theta)}{\rho}\left([1-\bar{\varphi}]^{\frac{\theta}{\theta-1}} + 1 - \bar{\varphi}\right)$$

while

$$n'_b(A_t) = \frac{-\varphi'(A_t)}{\rho} > 0$$

$$n_b''(A_t) = \frac{-\varphi''(A_t)}{\rho} < 0$$

$$\lim_{A_t \to +\infty} n_b(A_t) = \frac{1}{\rho} > \frac{2\gamma(1-\theta)}{\rho} \quad \text{and} \quad n_b(0) = \frac{1-\bar{\varphi}}{\rho} < n_a(0)$$

Moreover, the time devoted to the labor market of women is

$$\mathcal{L}(A_t) = \begin{cases} 1 - \rho n_a(A_t) - \varphi(A_t) & \text{if} \quad A_t \geq A^* \\ \\ 0 & \text{if} \quad A_t \leq A^* \end{cases} \tag{27}$$

with

$$\mathcal{L}'(A_t) = -\rho n_a'(A_t) - \varphi'(A_t) > 0 \quad \forall A_t \geq A^*$$

$$\mathcal{L}''(A_t) = -\rho n_a''(A_t) - \varphi''(A_t) < 0 \quad \forall A_t \geq A^*$$

and

$$\lim_{A_t \to +\infty} \mathcal{L}(A_t) = 1 - 2\gamma(1-\theta)$$

So Figure 7 below describing the impact of technology on fertility and female participation to the market.

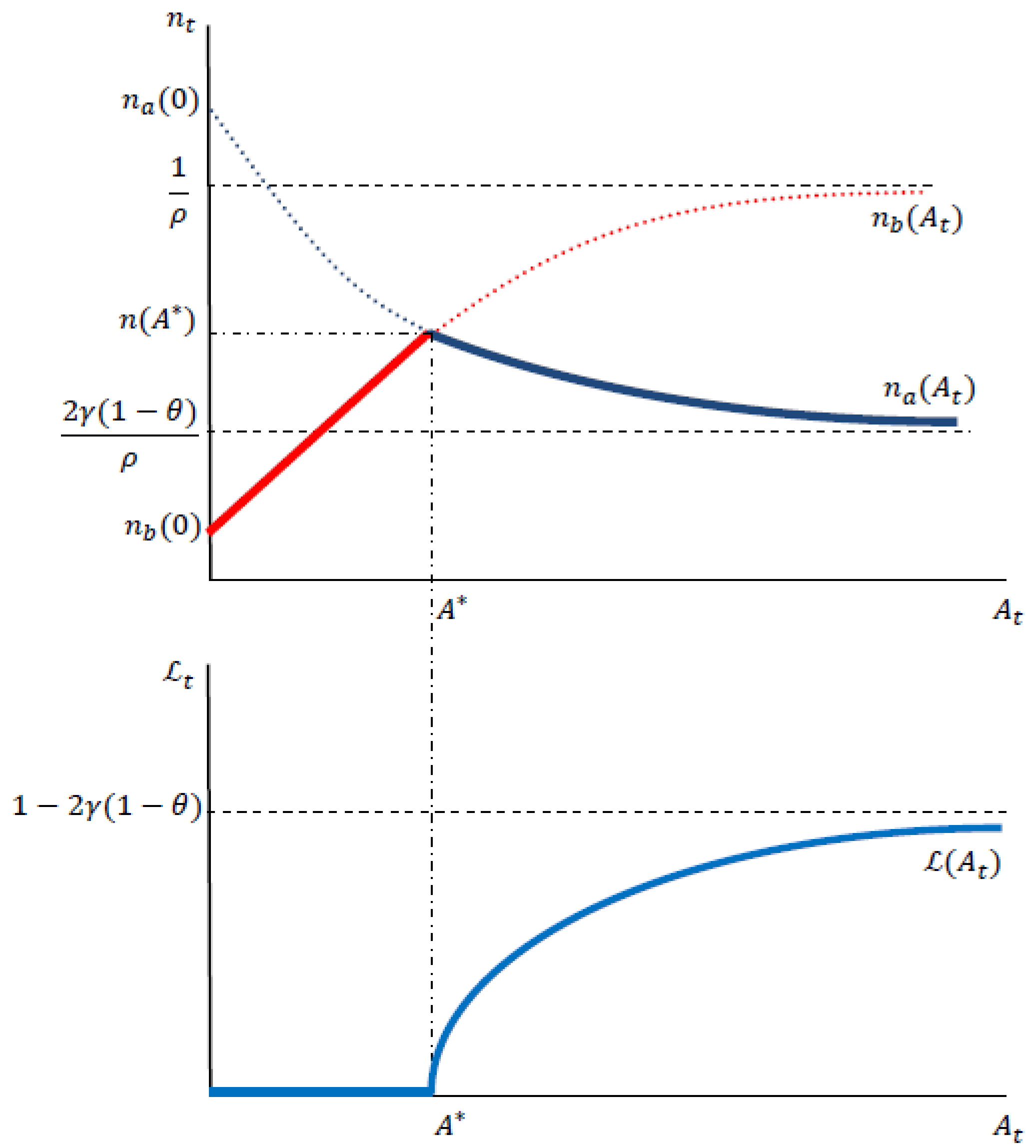

Fig 7. Fertility (upper graph) and female labor-force participation (lower graph) against technology

So, when the technological level is low enough (i.e. $A_t < A^*$), women spend full time doing housework and raising children. When the technological level increases but is still low, the time for doing housework decreases and the time for raising children increases, then in this case an increase in the technological level leads to an increase in fertility. When technological level exceeds the threshold A^*, women do housework and raise children part time, and participate the labor market in their remaining time. Technological progress makes the human capital gap between men and women decrease, as expressed in (24), and thus reduces the relative earning gap between men and women. This reduction in the relative earning gap implies an increase in the earnings of women. This increase in the earnings of women leads to an increase in the opportunity cost of raising children. From (21) we find that the educational investment for one couple of children increases in the potential earnings of women when the technological level is large enough (i.e. $A_t > A^*$). So when the cost of raising children physically increases, households will trade less quantity for higher quality children. Therefore, along with the decline in fertility and increase in educational investment due to the increase in technology, the labor-force participation of women increases.

5. Competitive equilibrium and Dynamics

The competitive equilibrium of the economy in period t is characterized by (i) the household's utility maximization under the constraints, (ii) the aggregate output equating the total return to human capital, (iii) the dynamics of the technological level, and (iv) the dynamics of population. Therefore, a competitive equilibrium $\{c_t, n_t, e^m_{t+1}, e^f_{t+1}, Y_t, w_t, A_{t+1}, L_{t+1}\}$ is determined by the following system of equations

$$c_t = \begin{cases} (1-\gamma)w_t[(e^m_t)^\theta + [1-\varphi(A_t)](e^f_t)^\theta] & \text{if} \quad A_t \geq A^* \\ \\ \frac{1-\gamma}{1-\gamma+\gamma\theta} w_t (e^m_t)^\theta & \text{if} \quad A_t \leq A^* \end{cases}$$

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left[\frac{(e^m_t)^\theta}{(e^f_t)^\theta} + 1 - \varphi(A_t)\right] & \text{if} \quad A_t \geq A^* \\ \\ \frac{1-\varphi(A_t)}{\rho} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$e^m_{t+1} = \begin{cases} \frac{\theta\rho w_t (e^f_t)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho w_t (e^m_t)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$e^f_{t+1} = \begin{cases} \frac{\theta\rho w_t (e^f_t)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho w_t (e^m_t)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$Y_t = f(A_t)L_t\left((e^m_t)^\theta + [1-\rho n_t - \varphi(A_t)](e^f_t)^\theta\right)$$

$$w_t = f(A_t)$$

$$A_{t+1} = \left[1 + g\left(\frac{(e^m_t)^\theta + (e^f_t)^\theta}{2}\right)\right] A_t$$

$$L_{t+1} = n_t L_t$$

for given A_t, L_t, e^m_t, e^f_t.

The competitive equilibrium can be fully characterized by the following reduced system describing the equilibrium dynamics of the level of technology A_{t+1} and educational investments for male and female children e^m_{t+1}, e^f_{t+1}.

$$A_{t+1} = \left[1 + g\left(\frac{(e^m_t)^\theta + (e^f_t)^\theta}{2}\right)\right] A_t \tag{28}$$

$$e^m_{t+1} = \begin{cases} \frac{\theta\rho f(A_t)(e^f_t)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho f(A_t)(e^m_t)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{29}$$

$$e^f_{t+1} = \begin{cases} \frac{\theta\rho f(A_t)(e^f_t)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho f(A_t)(e^m_t)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{30}$$

for a given initial condition A_0, e^m_0, and e^f_0.

To prove the convergence, which is stated in Proposition 2 below, of the dynamic system (28)-(30) , we have to prove the following Lemma 2.

Lemma 2: *For a dynamic equation $x_{t+1} = a_t x_t^\alpha$ with $\alpha \in (0,1)$, $x_0 > 0$, and $\lim_{t\to+\infty} a_t = a > 0$, then*

$$\lim_{t\to+\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

***Proof*:** See Appendix A3.

Proposition 2: *The overlapping generations economy as set up above, with any initial conditions $A_0 > 0$, e_0^m, and e_0^f, will converge to a regime of sustained growth characterized by a constant growth rate of technology, a constant fertility rate, constant education investments, and complete gender equality in education.*

Proof: In effect, for this economy, the level of technology increases unboundedly over time because the driving force for technological progress is positive investment in human capital. The technological progress appears even when the average human capital is very small. So it is straightforward from (21), (22), and the Lemma 2 that the educational investments will converge to a constant and equal level between male and female children. In particularly,

$$\bar{e} = \lim_{t\to+\infty} e_t^m = \lim_{t\to+\infty} e_t^f = \left[\frac{\theta\rho\bar{f}}{2(1-\theta)}\right]^{\frac{1}{1-\theta}}$$

From (26), the fertility rate will converge to a constant rate,

$$\bar{n} = \lim_{t\to+\infty} n_a(A_t) = \frac{2\gamma(1-\theta)}{\rho}$$

And the technology will grows at a constant rate

$$\bar{g} = g\left(\left[\frac{\theta\rho\bar{f}}{2(1-\theta)}\right]^{\frac{\theta}{1-\theta}}\right)$$

which are mentioned in the Proposition 2. Q.E.D.

The statement in the proposition 2 is rather consistent with stylized facts of the developed world where a modern sustained growth regime prevails characterized by unbounded economic growth, low and decreasing fertility rate, and high human capital.

6. Analysis

It would be interesting to link the theoretical results of this paper to the development process of western Europe characterized by three distinct regimes, (i) the Malthusian regime where both per-capita income and growth rate of population are very low; (ii) the demographic transition where both growth rate of population and per-capita income increase simultaneously; and later (iii) the modern sustained growth regime where the growth rate of population falls while per-capita income grows. During the development process of modern sustained growth, the labor-force participation of women increases along with a decline in gender inequalities in education and income.

Consider an economy in the early stage of development with very low initial level of technology and low human capital of both men and women. The technological level is low enough (i.e. $A_0 < A^*$) for women to have to spend their full time raising children and doing housework. The low technological level prevents women from participating in the labor market in two ways. First, it directly requires a large fraction of women's time to do housework. Second, it indirectly creates a gender inequality in education that makes women receive less educational investment from their parents, hence the opportunity cost of raising children physically becomes cheap, so that households prefer to increase their number of children rather than supplying the woman's labor to the market. Since, in this period, the income of a household is very low due to low human capital and low technological level then educational investments for their children are very small. Therefore, human capital are very low for both male and female children. Moreover, the growth rate of technology is very small as well because of low average human capital, which is the driving force for technological progress. Consequently, households invest very little education for their daughters because they anticipate the large fraction of time that their daughters have to devote to housework due to the low level of technology in the next period. Since men always supply their labor inelastically to the market then, while women have to spend a large fraction of their time for housework, households allocate a large fraction of education investment to their sons, making the education of their sons and daughters very unequal. In addition, in this stage of development, housework requires a very large fraction of women's time then the remaining time for raising children is very small, making the population growth rate low. Because the driving force of technological progress is the average human capital and technology grows even the average human capital is small, so, over time, technological level increases, while still being low (i.e. $A_t < A^*$), making the time for housework to decrease, and women to have more time to take care of children. In this stage, although the technological progress can increase the earnings of women, i.e. increase the opportunity cost of raising children physically, households increase the number of their children. Consequently, the fertility rate increases.

Over time, fertility increases along with technological progress and reaches the maximum when the technological level reaches A^*, generating a demographic transition. When the technological level is high enough (i.e. $A_t > A^*$), women will participate the labor market and their participation increases due to two effects of technology. First, the technological progress helps women save time in doing housework, leaving them more time for childcare and labor-force participation. Second, the technological progress improves the return to human capital, thus increases the earnings of women, meaning that the opportunity cost of raising children physically increases. The model shows that the fertility decreases due to the increase in the technological level. We also know the educational investment for one couple of children from (29) and (30) when $A_t > A^*$ that

$$e^m_{t+1} + e^f_{t+1} = \frac{\theta \rho f(A_t)(e^f_t)^\theta}{1-\theta}$$

i.e. this educational investment increases with respect to the level of technology. So, in this period, households trade low quantity for high quality of their children. The increases in the level of technology make households invest less education for male children to invest more education for female children because the return to female education is higher than that to male education. Consequently, the gender equality in education improves over time, accelerating the technological progress. This feedback loop puts the economy to enter the regime of sustained economic growth.

7. Conclusion

This paper develops a unified growth model capturing technological progress, gender inequality in education and fertility and the complex interaction between these issues to explain some stylized facts characterizing the development process. Particularly, the paper proposes a mechanism linking technology, gender inequality and fertility to shed a light that the transition from stagnation through demographic transition to modern sustained growth, along with the improvement in gender equality in education, income, as well as the increases in female labor-force participation are *inevitable* outcomes of development process when the driving force for technological progress is average human capital. The paper also shows that technological progress may increase female labor-force participation not only by liberating them from doing housework due to the appearance of time-saving household-sector products but also by leading households to reduce fertility due to the increase in the return of human capital, and hence increase in the cost of raising children. In addition, technological progress also makes households trade quantity for higher quality children.

Appendix

A1. Solving the household's optimization problem

$$\max_{\substack{c_t, n_t > 0 \\ e^m_{t+1}, e^f_{t+1} \geq 0}} \gamma \ln \left(n_t w_{t+1} \left[(e^m_{t+1})^\theta + [1 - \varphi(A_{t+1})](e^f_{t+1})^\theta \right] \right) + (1 - \gamma) \ln c_t$$

subject to

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) \leq w_t(h^m_t + [1 - \varphi(A_t)]h^f_t)$$

$$\rho n_t + \varphi(A_t) \leq 1$$

The Kuhn Tucker conditions are

$$\begin{pmatrix} \frac{1-\gamma}{c_t} \\ \frac{\gamma}{n_t} \\ \frac{\gamma\theta(e^m_{t+1})^{\theta-1}}{(e^m_{t+1})^\theta + [1-\varphi(A_{t+1})](e^f_{t+1})^\theta} \\ \frac{\gamma[1-\varphi(A_{t+1})]\theta(e^f_{t+1})^{\theta-1}}{(e^m_{t+1})^\theta + [1-\varphi(A_{t+1})](e^f_{t+1})^\theta} \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ \rho w_t h^f_t + e^m_{t+1} + e^f_{t+1} \\ n_t \\ n_t \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$+\lambda_3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \lambda_4 \begin{pmatrix} 0 \\ \rho \\ 0 \\ 0 \end{pmatrix}$$

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) - w_t(h^m_t + [1 - \varphi(A_t)]h^f_t) \leq 0$$

$$-e_{t+1}^m \leq 0$$

$$-e_{t+1}^f \leq 0$$

$$\rho n_t + \varphi(A_t) - 1 \leq 0$$

$$\lambda_1 \left[c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1-\varphi(A_t)]h_t^f) \right] = 0$$

$$\lambda_2 e_{t+1}^m = 0$$

$$\lambda_3 e_{t+1}^f = 0$$

$$\lambda_4[\rho n_t + \varphi(A_t) - 1] = 0$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$$

For these conditions, it is straightforward to show that $\lambda_1 > 0$, because

$$\lambda_1 = \frac{1-\gamma}{c_t} \neq 0$$

since $\gamma \in (0,1)$ and $c_t > 0$. That is to say the budget constraint is binding.

Since $\theta - 1 < 0$, then it is also straightforward from conditions

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_2$$

$$\frac{\gamma[1-\varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_3$$

that $e_{t+1}^m > 0$ and $e_{t+1}^f > 0$ to guarantee the left-hand-sides (both numerators and denominators) to be determined. Therefore, $\lambda_2 = \lambda_3 = 0$. That is to say, the positivity constraints of education investments for male and female children are never binding at the optimal solution.

Now we consider two cases: (i) the time constraint of the woman is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, then $\lambda_4 = 0$, so that

$$\frac{1-\gamma}{c_t} = \lambda_1 > 0$$

$$\frac{\gamma}{n_t} = \frac{1-\gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \tag{31}$$

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t} n_t \tag{32}$$

$$\frac{\gamma[1-\varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t} n_t \tag{33}$$

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1-\varphi(A_t)]h_t^f) = 0 \tag{34}$$

From (32) and (33) we have

$$\frac{(e_{t+1}^m)^{\theta-1}}{[1-\varphi(A_{t+1})](e_{t+1}^f)^{\theta-1}} = 1$$

$$\Rightarrow \quad e_{t+1}^f = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} e_{t+1}^m \tag{35}$$

From (31) we have

$$\frac{1-\gamma}{c_t} n_t = \frac{\gamma}{\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f} \tag{36}$$

Substitute (35) and (36) into (32) we have

$$\frac{\theta}{(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m} = \frac{1}{\rho w_t h_t^f + (1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m}$$

So, we obtain

$$e_{t+1}^m = \frac{\theta \rho w_t h_t^f}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{37}$$

Hence,

$$e_{t+1}^f = \frac{\theta \rho w_t h_t^f}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{38}$$

From (31), (34), (37) and (38) we have

$$\frac{1}{\gamma}\left(\rho w_t h_t^f + \frac{\theta \rho w_t h_t^f}{1-\theta}\right) n_t = w_t(h_t^m + [1-\varphi(A_t)]h_t^f)$$

Hence,

$$n_t = \frac{\gamma(1-\theta)}{\rho}\left(\frac{h_t^m}{h_t^f} + 1 - \varphi(A_t)\right) \tag{39}$$

and

$$c_t = (1-\gamma)w_t(h_t^m + [1-\varphi(A_t)]h_t^f) \tag{40}$$

So, the solution is characterized by four equations (37)-(40).

- (ii) If $\rho n_t + \varphi(A_t) = 1$, so that

$$\frac{1-\gamma}{c_t} = \lambda_1 > 0$$

$$n_t = \frac{1-\varphi(A_t)}{\rho} \tag{41}$$

$$\frac{\gamma\rho}{1-\varphi(A_t)} = \frac{1-\gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) + \lambda_4 \rho \tag{42}$$

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho} \tag{43}$$

$$\frac{\gamma[1-\varphi(A_{t+1})]\theta(e_f^{t+1})^{\theta-1}}{(e_{t+1}^m)^\theta+[1-\varphi(A_{t+1})](e_{t+1}^f)^\theta}=\frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho} \tag{44}$$

$$c_t+\frac{1-\varphi(A_t)}{\rho}(\rho w_t h_t^f+e_{t+1}^m+e_{t+1}^f)-w_t(h_t^m+[1-\varphi(A_t)]h_t^f)=0 \tag{45}$$

From (43) and (44) we have

$$\frac{(e_{t+1}^m)^{\theta-1}}{[1-\varphi(A_{t+1})](e_{t+1}^f)^{\theta-1}}=1$$

$$\Rightarrow \quad e_{t+1}^f=[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}}e_{t+1}^m \tag{46}$$

Substitute (46) into (43) we have

$$\frac{\gamma\theta}{(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m}=\frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho}$$

$$\Rightarrow \quad c_t=\frac{1-\gamma}{\gamma\theta}\frac{1-\varphi(A_t)}{\rho}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m \tag{47}$$

Substitute (46) and (47) into (45) we have

$$\frac{1-\varphi(A_t)}{\rho}\times$$

$$\left\{\frac{1-\gamma}{\gamma\theta}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m+\rho w_t h_t^f+(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m\right\}$$

$$=w_t(h_t^m+[1-\varphi(A_t)]h_t^f)$$

$$\Leftrightarrow \frac{1-\gamma+\gamma\theta}{\gamma\theta}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m=\frac{\rho w_t h_t^m}{1-\varphi(A_t)}$$

So, we obtain

$$e_{t+1}^m=\frac{\gamma\theta\rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{48}$$

Hence,

$$e^f_{t+1} = \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{49}$$

Substitute (48) into (47) we have

$$c_t = \frac{(1-\gamma)w_t h^m_t}{1-\gamma+\gamma\theta} \tag{50}$$

Finally, substitute (48), (49), and (50) into (42) we have

$$\lambda_4 = \frac{\gamma(1-\theta)}{1-\varphi(A_t)} - \frac{h^f_t}{h^m_t}(1-\gamma+\gamma\theta) \tag{51}$$

Hence, in this case, the optimal solution is

$$c_t = \frac{(1-\gamma)w_t h^m_t}{1-\gamma+\gamma\theta}$$

$$n_t = \frac{1-\varphi(A_t)}{\rho}$$

$$e^m_{t+1} = \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})}$$

$$e^f_{t+1} = \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})}$$

A2. Checking the SOCs for the maximization problem of household

Since the opitimization problem is not convex, then for the FOCs to be sufficient conditions to characterize a (local) maximum to the optimization problem, we have to check the sufficient SOCs. We know from Appedix A5.1 that the positivity constraints of education investments for male and female children are never binding at the solution, while the budget constrant is always binding at the solution, and the time constraint of women can be binding or nonbinding. So we have to check the bordered Hessian matrix in two cases: (i) the time constraint of women is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^i = \begin{pmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{pmatrix}$$

where B, C, and D are defined below (note that for lightening notations, we denote $e^m_{t+1} = e_m$, $e^f_{t+1} = e_f$, and $\varphi(A_{t+1}) = \varphi$).

$$\begin{aligned} B = \frac{\partial^2 u_t}{\partial e_m^2} &= \gamma\theta \frac{(\theta-1)e_m^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - \theta e_m^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} \\ &= \frac{-\gamma\theta e_m^{\theta-2}\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0 \end{aligned}$$

$$\begin{aligned} D = \frac{\partial^2 u_t}{\partial e_f^2} &= \gamma\theta(1-\varphi) \frac{(\theta-1)e_f^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - (1-\varphi)\theta e_f^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} \\ &= \frac{-\gamma\theta(1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0 \end{aligned}$$

$$C = \frac{\partial^2 u_t}{\partial e_m \partial e_f} = \frac{-\gamma\theta^2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$E = \rho w_t h_t^f + e_m + e_f > 0$$

Now we prove two following properties:

Property 1: $BD > C^2$.
In effect, $BD > 0$, $C^2 > 0$, and

$$\frac{BD}{C^2} = \frac{\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\theta^2(1-\varphi)e_m^\theta e_f^\theta}$$

$$= \frac{(1-\theta)e_m^{2\theta} + [(1-\theta)^2+1](1-\varphi)e_m^{\theta}e_f^{\theta} + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^{\theta}e_f^{\theta}}$$

$$= \frac{(1-\theta)e_m^{2\theta} + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^{\theta}e_f^{\theta}} + \frac{2(1-\theta)}{\theta^2} + 1 > 1$$

i.e. $BD > C^2$ ■

Property 2: $2C > B + D$.

We know that $C < 0$, $B < 0$, and $D < 0$, then the property 2 is equivalent to

$$\frac{B+D}{2C} > 1$$

In effect,

$$\frac{B+D}{2C} =$$

$$\frac{e_m^{\theta-2}\left[e_m^{\theta} + (1-\theta)(1-\varphi)e_f^{\theta}\right] + (1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^{\theta} + (1-\varphi)e_f^{\theta}\right]}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}$$

$$= \frac{e_m^{2\theta-2} + (1-\varphi)^2 e_f^{2\theta-2}}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}} + V$$

where $V = \frac{(1-\theta)[e_f^{\theta}e_m^{\theta-2} + e_f^{\theta-2}e_m^{\theta}]}{2\theta e_m^{\theta-1}e_f^{\theta-1}} > 0$.
Applying the trivial inequality, $e_m^{2\theta-2} + (1-\varphi)^2 e_f^{2\theta-2} \geq 2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}$, we have

$$\frac{B+D}{2C} \geq \frac{1}{\theta} + V > 1$$

i.e. $2C > B + D$ ■

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_2^i\right| = \begin{vmatrix} 0 & 1 & E \\ 1 & \frac{\gamma-1}{c_t^2} & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \frac{1-\gamma}{c_t^2}E^2 + \frac{\gamma}{n_t^2} > 0$$

$$\left|\bar{H}_3^i\right| = \begin{vmatrix} 0 & 1 & E & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & B \end{vmatrix} = B\left|\bar{H}_2^i\right| + \frac{(\gamma-1)\gamma}{c_t^2} < 0$$

$$\left|\bar{H}_4^i\right| = \left|\bar{H}^i\right| = \begin{vmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & E & n_t & n_t \\ 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ 0 & 0 & B & C \\ 0 & 0 & C & D \end{vmatrix} + \frac{\gamma-1}{c_t^2}\begin{vmatrix} 0 & E & n_t & n_t \\ E & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix}$$

$$= \left[\frac{\gamma}{n_t^2} + \frac{1-\gamma}{c_t^2}E^2\right](BD - C^2) + \frac{(1-\gamma)\gamma}{c_t^2}(2C - B - D) > 0$$

under properties 1 and 2 above.
So the solution to the agent's problem in this case is a maximum indeed.

- (ii) If $\rho n_t + \varphi(A_t) = 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^{ii} = \begin{pmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{pmatrix}$$

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_3^{ii}\right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t \\ 0 & 0 & 0 & \rho & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & 0 & B \end{vmatrix}$$

$$= B\rho^2 - n_t^2 \begin{vmatrix} 0 & 0 & \rho \\ 0 & \frac{\gamma-1}{c_t^2} & 0 \\ \rho & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \left(B - n_t^2 \frac{1-\gamma}{c_t^2}\right)\rho^2 < 0$$

$$\left|\bar{H}_4^{ii}\right| = \left|\bar{H}^{ii}\right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{vmatrix}$$

$$= -\rho^2 \begin{vmatrix} 0 & 1 & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix} = \rho^2 \begin{vmatrix} 1 & n_t & n_t \\ 0 & B & C \\ 0 & C & D \end{vmatrix} + \rho^2 \frac{1-\gamma}{c_t^2} \begin{vmatrix} 0 & n_t & n_t \\ n_t & B & C \\ n_t & C & D \end{vmatrix}$$

$$= \rho^2 \left[BD - C^2 + \frac{1-\gamma}{c_t^2}(2C - B - D)\right] > 0$$

under properties 1 and 2 above.
So the solution to the agent's problem in this case is also a maximum indeed.

A3. Proof of Lemma 2.

In effect, since $\lim_{t\to+\infty} a_t = a > 0$ then $\forall \varepsilon \in (0, a)$, $\exists T_0$ such that $\forall t \geq T_0$, we have

$$a - \varepsilon \leq a_t \leq a + \varepsilon$$

Define

$$X_0 = Y_0 = Z_0 = x_{T_0}$$

and

$$X_{t+1} = a_{T_0+t} X_t^{\alpha}, \quad Y_{t+1} = (a+\varepsilon) Y_t^{\alpha}, \quad Z_{t+1} = (a-\varepsilon) Z_t^{\alpha}$$

We know that

$$\lim_{t \to +\infty} Y_t = (a+\varepsilon)^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad \lim_{t \to +\infty} Z_t = (a-\varepsilon)^{\frac{1}{1-\alpha}}$$

We also have

$$Z_1 = (a-\varepsilon) X_0^{\alpha} \le X_1 = a_{T_0} X_0^{\alpha} \le (a+\varepsilon) X_0^{\alpha} = Y_1$$

and

$$Z_2 = (a-\varepsilon) Z_1^{\alpha} \le (a-\varepsilon) X_1^{\alpha} \le X_2 = a_{T_0+1} X_1^{\alpha} \le (a+\varepsilon) X_1^{\alpha} \le (a+\varepsilon) Y_1^{\alpha} = Y_2$$

$$\vdots$$

and so on, by induction we have

$$Z_t \le X_t \le Y_t, \ \forall t.$$

Hence,

$$(a-\varepsilon)^{\frac{1}{1-\alpha}} = \lim_{t \to +\infty} Z_t \le \lim_{T \to +\infty} \left(\inf_{t \ge T} X_t \right) \le \lim_{T \to +\infty} \left(\sup_{t \ge T} X_t \right) \le \lim_{t \to +\infty} Y_t = (a+\varepsilon)^{\frac{1}{1-\alpha}}$$

That is to say

$$(a-\varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq (a+\varepsilon)^{\frac{1}{1-\alpha}},\ \forall\varepsilon \in (0,a)$$

Hence,

$$\lim_{\varepsilon\to0^+}(a-\varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq \lim_{\varepsilon\to0^+}(a+\varepsilon)^{\frac{1}{1-\alpha}}$$

i.e.

$$a^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq a^{\frac{1}{1-\alpha}}$$

which implies

$$\lim_{t\to+\infty} X_t = a^{\frac{1}{1-\alpha}}$$

i.e.

$$\lim_{t\to+\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

Q.E.D.

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